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The Correlation of anatomical or physiological measurements

[From The American Anthropologist for July, 1894.]

THE CORRELATION OF ANATOMICAL OR PHYSIOLOGICAL MEASUREMENTS.

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It is one of the objects of anthropometric investigations to establish types of certain varieties of man, the results of anthropometric statistics being a means of describing in exact terms a certain variety and its variability. This method of describing varieties has been applied so far in the case of man and of domesticated animals only. In most cases biologists have been satisfied with verbal descriptions of varieties and of variability and no attempts at an exact definition have been made. It is clear, however, that the method may be applied advantageously in all investigations of variation and a *biometric* method would undoubtedly open new ways of attacking the problems of variation and transformation.

The results of measurements of a certain variety present themselves in the following way: The measurement has a series of values which occur with different frequency. Generally the average of all its values is found most frequently and the other values are distributed around it according to the laws of chance. Then the description of the type is an enumeration of those distributions as studied in a series of measurements. The measurements may relate to anatomical features as well as to physiological functions or to relations between the two.

The problem of defining the type is, however, not finally solved by the enumeration. The measurements ought to be selected and treated in such a manner that those which are independent of each other may be recognized as such, and that typical correlations be brought out clearly.

It may be well to state this problem in a mathematical form which will bring out most clearly the points to be observed.

Any anatomical or physiological measurement of an organism may be considered a function of the general conditions of heredity and environment affecting the measured individual as a whole and in those parts which have been subjected to measure-

ment. If two such measurements be called M and M_1, the conditions $a_1, a_2, a_3 \ . \ . \ . \ a_n$; we have

$$M = f\,(a_1, a_2, a_3 \ . \ . \ . \ a_q);$$
$$M = f_1\,(a_1, a_2, a_3 \ . \ . \ . \ a_q).$$

The variable conditions $a_1, a_2 \ . \ . \ . \ . \ a_q$ may be divided into three classes: those which influence M alone, which we will call x; those which influence M_1 alone, which we will call y; and those which influence both M and M_1, which we will call z.

$$(1) \quad M = f\,(x_1, x_2 \ . \ . \ . \ x_m;\ z_1, z_2 \ . \ . \ . \ z_p);$$
$$M_1 = f_1\,(y_1, y_2 \ . \ . \ . \ y_n;\ z_1, z_2 \ . \ . \ . \ z_p).$$

When the influence of z disappears in these functions M is independent of M_1, and both ought to be contained in the list of measurements, but their proportion would not indicate any biological law.

Biological considerations lead to the conclusion that in most cases the influence of z will be small as compared to that of x and y. When two measurements of distinct parts of the body are compared the complexes of cells which compose these parts have distinct life histories, and have, therefore, been subject to quite different influences. The same may be said of neighboring measurements taken in different directions—for instance, transversal and longitudinal measurements. Thus when breadth of head and stature are compared, we know that after the fifth year of life the rate of increase of the former is very slight, while stature continues to increase very rapidly during the whole period of growth. Circumstances which may exert an influence upon the latter will, therefore, remain almost without effect upon the former. While it is probable that there are always causes which affect only the one or the other measurement, their amount as compared to the causes affecting both simultaneously must be left to an investigation of the observed measurement.

This inquiry may be carried on in the following manner: Supposing that the influence of z disappears entirely, we have

$$M = f\,(x_1, x_2 \ . \ . \ . \ x_m);$$
$$M_1 = f_1\,(y_1, y_2 \ . \ . \ . \ y_n).$$

If we select a group of individuals for which $M_1 = constans$, $x_1, x_2 \ldots x_m$ are not restricted in any way, and the distribution of M will, therefore, remain unaffected by the restriction in regard to M_1. In most cases the distribution of M will follow the laws of chance with greater or lesser approximation, and the values of M may be represented by their average. Then the average will remain constant whatever the value of M_1 may be. This consideration may be inverted and we may say that whatever the value of M, the correlated average of M_1 will remain the same.

When the influence of z is small as compared to those of x and y respectively, a different phenomenon will follow. The equation (1) may be transformed as follows:

$$(2) \quad M = F(x_1, x_2 \ldots x_m) + F_1(z_1, z_2 \ldots z_p) + F_2(x_1, x_2 \ldots x_m; z_1, z_2 \ldots z_p);$$

$$M_1 = \Phi(y_1, y_2 \ldots y_n) + \Phi_1(z_1, z_2 \ldots z_p) + \Phi_2(y_1, y_2 \ldots y_n; z_1, z_2 \ldots z_p).$$

According to our assumption, F_1, F_2, Φ_1, and Φ_2 are small as compared to F and Φ. That means that when we select a group of individuals for which $M_1 = constans$, we shall find that the corresponding value of M is slightly modified, and consequently the average value of all the M belonging to $M_1 = constans$ will also be slightly modified. The modification will be the greater the greater the influence of z. *Vice versa*, when M is selected as constant, M_1 will be modified only slightly.

When F, F_2, Φ, and Φ_2 are small as compared to F_1 and Φ_1, a similar result may be expected. The proof of this is not quite easy, but I will introduce a few considerations which will serve to illustrate the argument. Provided the functions Φ_1 and F_1 are entirely independent of each other. When M_1 differs considerably from the average, $z_1, z_2 \ldots z_p$ must fill certain conditions which will always give Φ_1 a large value. There are numerous combinations of $z_1, z_2 \ldots z_p$ possible which will fill these conditions. As there is no connection between F_1 and Φ_1, and as the values of both functions, which are near the general average, are more frequent than the higher or lower values, the majority of the values of F_1 must be near the average. The average of all the F_1 must, therefore, be near the general average.

When, however, a certain series of causes control two measurements, it seems very probable that they act upon both in the same manner; that is to say, if the averages of F_1 and of Φ_1 be called μ' and μ_1' respectively,

$$F_1 : \Phi_1 = \mu' : \mu_1'.$$

When $M_1 = constans$, and the influence of y disappears, the deviation from the general average will be borne by z alone. When the influence of y increases, the deviation will be borne by both z and y, because the general deviation will result most frequently from smaller deviations caused by both variables. We will call the component parts of this deviation d and d', and the average of all the F, μ; that of all the Φ, μ_1.

$$M_1 = (\mu_1 + d) + (\mu_1' + d') = \mu_1 + \mu_1' + d + d'.$$

Then, on account of the relation of F_1 and Φ_1, we have

$$M = \mu + (\mu_1' + d')\frac{\mu'}{\mu_1'} = \mu + \mu' + d'\frac{\mu'}{\mu_1'}.$$

This equation proves that in this case also the increase of M is less proportionately than that of M_1.

I will give a few illustrations of this phenomenon.

A comparison of the correlation of length and breadth of head of 923 adult male Sioux, Crow, and western Ojibwa gives the following result:

Group	I.	II.	III.	IV.	V.	VI.
Length of head	180–184*	185–189	190–194	195–199	200–204	205–209*
Average length of head	182.2	187.1	192.0	196.7	201.3	205.9
Average breadth of head	153.8	153.8	154.8	156.2	157.8	159.4

When the same individuals are classed according to breadth of head, the following results are obtained:

Group	I.	II.	III.	IV.	V.
Breadth of head	145–149*	150–154	155–159	160–164	165–169*
Average breadth of head	147.2	152.2	156.8	161.3	166.2
Average length of head	190.9	193.5	195.5	196.4	199.1

*Millimeters.

I have arranged these results graphically on Fig. 1, in which the scale of breadth of head is selected so that it is proportional to the proportion of the averages of length and breadth of head. This is done in order to make the curves strictly comparable.

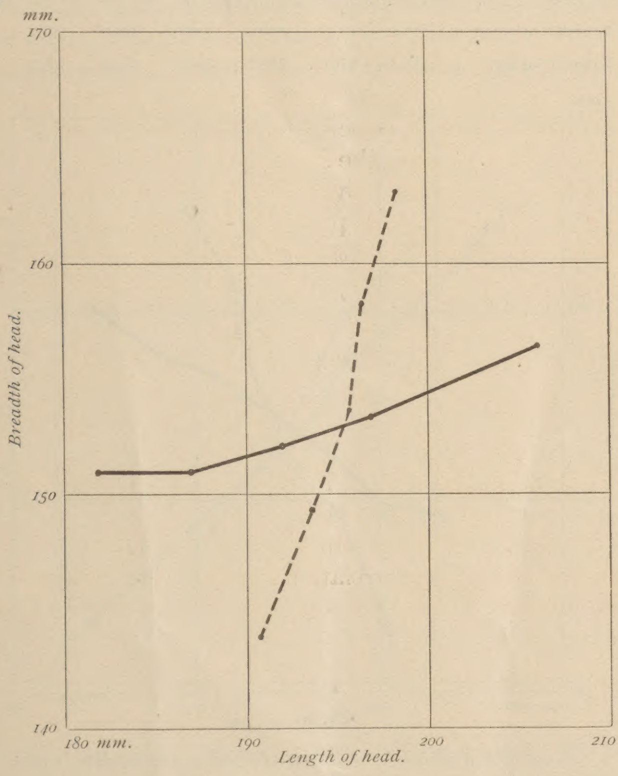

FIG. 1.—Correlation between length of head and breadth of head. 923 Indians. (——— Breadth determined by length; ------- Length determined by breadth.)

It is clear that if the breadth of head were a complete function of the length of head there could be only one curve expressing the interrelation between the two measurements. The fact that there are two curves shows clearly that the one measurement does not define completely the other, but that a number of factors influence each by itself.

The following table gives a comparison between breadth of head and breadth of face among 782 adult male Sioux and Crow (Fig. 2):

Group	I.	II.	III.	IV.	V.	VI.	VII.
Breadth of head	144–148*	149–151	152–154	155–157	158–160	161–163	164–168*
Average breadth of head	146.8	150.3	153.0	156.0	159.0	161.9	165.5
Average breadth of face	144.5	146.0	148.4	150.6	152.4	153.5	156.1

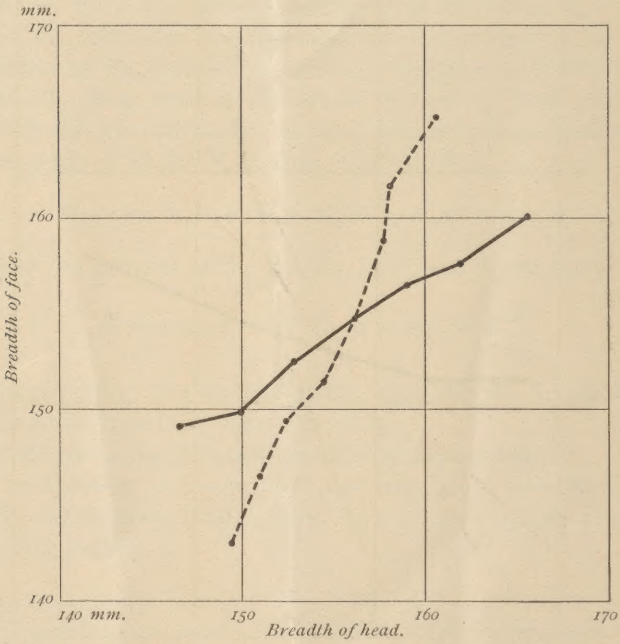

FIG. 2.—Correlation between breadth of head and breadth of face. 782 Indians. (——— Face determined by head; ------- Head determined by face.)

When the same individuals are classed according to breadth of face, we have:

Group	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Breadth of face	136–140*	141–143	144–146	147–149	150-152	153–155	156–158	159–164*
Average breadth of face	138.9	142.2	145.1	148.0	150.8	153.9	156 9	160.4
Average breadth of head	149.7	151.1	152.4	154.4	156.3	157.8	159.5	161.7

* Millimeters.

Finally, I will compare stature and finger-reach of 801 adult male Sioux, Crow, and western Ojibwa between 20 and 60 years of age:

Group	I.	II.	III.	IV.	V.	VI.	VII.
Stature	161–164*	165–168	169–172	173–176	177–180	181–184	185–188*
Average stature	163.2	167.0	171.1	174.9	178.7	182.7	186.7
Average finger-reach	172.1	175.7	179.6	182.9	186.8	191.4	196.0

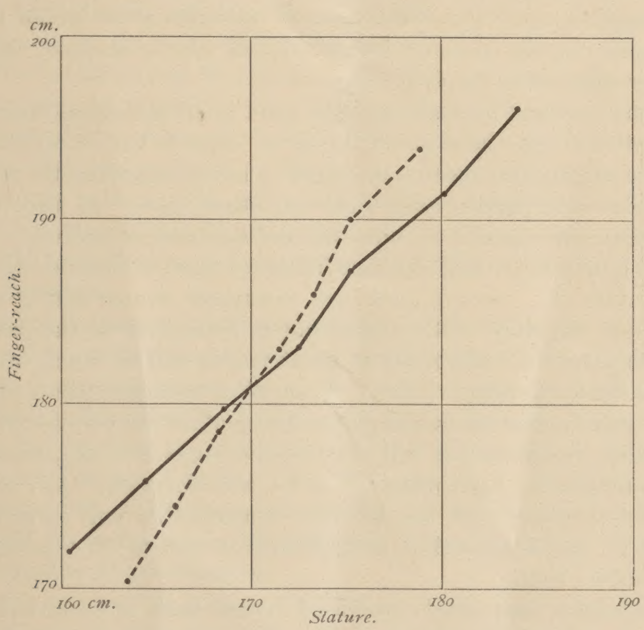

FIG. 3.—Correlation between stature and finger-reach. 801 Indians. (——— Finger-reach determined by stature; -------. Stature determined by finger-reach.)

Classed according to finger-reach:

Group	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
Finger-reach	164–167†	168–171	172–175	176–179	180–183	184–187	188–191	192–195†
Average finger-reach	166.8	170.2	174.1	178.1	182.0	185.7	189.7	193.7
Average stature	162.3	165.5	168.2	170.6	173.3	175.5	177.6	181.3

~~* Millimeters.~~ * † Centimeters.

I have selected these three pairs of measurements in order to illustrate the varying degrees of correlation. It is clear that the correlation of the first pair is very slight, while that of the last pair is very strong—that is to say, the influence of z is very slight in the first pair and very strong in the last pair. We find what was expected from a consideration of equation (2), that—

When any two biological measurements are considered as correlated and individuals showing a certain value of the first measurement are grouped together; then the average of the values of the second measurement for this group of individuals will also be changed, but to a lesser degree than the first.

This law will be found to hold good in all biological measurements. It has been pointed out as regards the correlation of brain weight and stature or weight of body, regarding the size of the foot and stature, and in several other cases. It also holds good in correlations of functions and anatomical features.

We may draw still another inference from a consideration of equation (1). As x, y, and z are fortuitous causes acting upon M and M_1, they will be distributed according to the laws of chance, and the frequency of great values will be much smaller than that of average values. When M_1 is constant, y and z must fill certain conditions and only a group of all the possible values of these two variables will be available; that is to say, when M_1 is constant, the variability of y and z must decrease. As z affects also the values of M, the decrease in variability will make itself felt in the distribution of that measurement. We conclude, therefore, that—

Whenever individuals showing a certain value of a measurement are grouped together, the variability of any second measurement of the group is smaller than the variability of the whole series.

The smaller the influence of z as compared to that of x, the less the variability will be affected, and we may consider the amount of the decrease in variability a supplementary measure of the proportion between the influences of x and z or a measure of the amount of correlation between the two measurements. To give an example: In the case of the correlation between length and breadth of head the decrease of variability is very slight. In the case of correlation between breadth of head and breadth of face we find a decrease of from about 5.5 mm. to 4.5 mm.; in that of correlation between stature, and finger-reach of from 6.3

cm. to 3.7 cm. It will be seen that the nearer the two curves representing the correlated values, the greater the decrease in variability. The decrease is, however, always small.

It can be shown in what cases this is due to the fact that the two measurements are influenced by independent causes, x and y, and when it is due to restrictions in regard to the common causes, z. When M_1 differs much from the average, y and z must also differ much from the average, and certain groups of possible values of these variables do not bring about the desired effect. Therefore the greater M_1 the more restricted the values of z and y. Therefore the variability of M must decrease with increasing M_1 unless the unchanged variability of x obscures this influence. I have not been able so far to find any M the variability of which decreases with increasing M_1, so that I conclude that in most cases there must exist a great many causes which influence the two measurements independently of each other. This fact shows that it would prove futile to endeavor to discover the ultimate causes of correlations.

The restrictions of z which result from the selection of a constant value of M_1 will also affect the distribution of the values of F_1 and F_2 and consequently those of M. It is very probable that they will not be distributed according to the laws of chance. As no decrease in the variability of M with increasing M_1 has been observed yet, it is doubtful if this influence makes itself felt materially. This consideration has led me, however, to investigate the influences of distribution in groups to that of the total number of observations.

As the value of M increases with increasing M_1 the distribution of cases is such that with increasing M_1 the maximum of frequency of M moves to higher and higher values. The general distribution will be the resultant of the superposition of all these single distributions. It follows from this fact that there must be a tendency to produce probability curves in the grand total, even if the component distributions show considerable deviations from this law. Whenever, therefore, there is any suspicion of such a deviation it may be investigated profitably by grouping the observed individuals. I have carried out this attempt for the correlation breadth of face and breadth of head among male adult half-blood Indians and found the expected result. The breadth of face of half-bloods shows the phenomenon that there is a slight deviation from the probability curve,

which seems to be sufficient, however, to lead to the conclusion that there is a cause for it. In the following table, which represents measurements of 377 adult males, it will be observed that the frequency of the measurement 144–145 mm. is not as frequent as the neighboring measurements. As the head measurements resulting from similar numbers of observations are arranged much more regularly, I expressed on a former occasion the conclusion that we find here the narrow faces of the white and the broad faces of the European preserved in the half-bloods; that there is no tendency to reproduce a middle form, but a tendency to preserve one of the parental forms. It appears that when the material is subdivided according to breadth of head the same phenomenon appears with much greater force in each of these groups, thus proving beyond a doubt that the former interpretation is correct. It might be said that the same result would appear as clearly when the proportion of breadth of face and breadth of head were tabulated. This is undoubtedly true, but proportions of this character also change their relations with increasing absolute values of the measurement, so that the subdivision will bring out the phenomenon more clearly than the index. Besides, its repeated occurrence in four distinct series is the strongest proof of its reality.

Breadth of Face of Half-blood Indians as Determined by Breadth of Head.

Group	I	II	III	IV	V
Breadth of head (mm.)	149–151	152–154	155–157	158-160	All individuals observed.
Breadth of face.	Frequency of occurrence (in %).				
130-131 mm	0.5	....	0.4	....	0.4
132–133	2.7	....	1.5	1.2	1.2
134–135	5.0	2.0	1.5	0.6	2.3
136-137	7.8	4.6	2.3	1.2	5.0
138–139	20.6	9.8	3.1	4.0	9.9
140–141	22.3	16.9	5.7	5.7	12.1
142–143	8.9	16.9	18.8	13.8	14.1
144–145	13.4	12.6	17.3	13.8	12.3
146–147	10.6	17.3	21.5	10.9	13.3
148-149	3.3	11.0	12.3	17.8	10.3

Breadth of Face of Half-blood Indians, etc.—Continued.

150–151..............	1.7	5.1	7.7	12.7	7.7
152–153..............	2.7	3.1	5.4	5.7	5.3
154–155..............	0.5	0.7	1.9	2.3	1.7
156–157..............	....	....	0.8	3.5	1.4
158–159..............	....	....	....	4.0	1.4
160–161..............	....	....	....	1.2	0.2
162–163..............	....	....	....	....	0.4
164–165..............	....	....	....	1.2	0.5
166–167..............	....	....	....	0.6	0.1
168–169..............	....	....	....	....	....

It will be remarked that in the preceding table the first maximum is strongest for the lesser breadths of head, while the second maximum is stronger for the higher values of breadth of head.

I was much surprised at finding a similar phenomenon when tabulating the length of head of the Sioux, Crow, and western Ojibwa in relation to their statures. While the general distribution shows hardly any asymmetry, the distribution of length of head arranged according to stature shows decided asymmetries of such a character that I am assured that these tribes are mixed of two elements, one having an average length of head of 193 mm., the other of 197 mm. It will be seen that two of these columns have only one decided maximum. These columns are, however, so asymmetrical and the asymmetry fits in so well with the preceding maxima that we are justified in considering this fact a corroboration of the evidence of the preceding columns.

Length of Head of Sioux, Crow, and Western Ojibwa as Determined by Stature.

Group.......	I	II	III	IV	V	VI
Stature (cm.)	160–164	165–169	170–174	175–179	180–184	185–189
Length of head.	Frequency of occurrence (in %).					
170–171 mm..	....	....	....	....	....	....
172–173......	1.2	....	....	....	....	....
174–175......	1.7	....	....	....	....	....
176–177......	0.6	....	0.2	....	....	....
178–179......	1.7	0.5	0.4	0.7	....	....

Length of Head of Sioux, Crow, and Western Ojibwa—Continued.

180–181......	4.6	1.8	0.9	1.1	0.5	....
182–183......	5.2	2.1	1.6	1.1	0.5	....
184–185......	8.6	4.6	2.6	4.0	1.8	0.9
186–187......	11.5	5.7	4.8	3.4	3.1	1.8
188–189......	4.0	9.3	7.6	6.9	1.8	7.2
190–191......	8.1	14.2	11.7	9.5	6.2	13.5
192–193......	12.7	9.8	12.5	9.5	12.9	12.7
194–195......	17.2	11.6	12.3	13.2	15.3	14.4
196–197......	7.5	14.5	13.5	16.6	14.5	10.0
198–199......	2.3	8.8	11.4	13.4	12.9	14.4
200–201......	4.0	8.0	7.2	6.9	12.7	11.8
202–203......	3.5	3.1	5.0	4.6	7.8	2.7
204–205......	2.3	3.1	3.2	4.2	5.9	3.6
206–207......	2.9	1.8	2.2	2.8	3.4	4.5
208–209......	0.6	0.8	1.5	0.9	0.3	2.7
210–211......	....	0.3	1.3	1.1	0.5	....
212–213......	....	....	....	0.2	....	....